

## Valitiy of Multiconductor Transmission Line Model (MTL) in Analysis and Design of Grounding Grids Buried in Lossy Frequency-Dependent Ground

Seyyed Sajjad Sajjadi  
MSC student from Arak University

Sajjad Mehrabi  
MSC student from Arak University

Saeed Reza Ostadzadeh<sup>1</sup>  
Assistant professor from Arak University

### Abstract

In this paper, we propose an approximate model called multi-conductor transmission line model (MTL) for transient analyses of grounding grid buried in dispersive grounds. The grounding grid is corner and center-subjected to lightning current. The simulation results via this model are in good agreement with full-wave methods in previously published papers. In addition, the run-time using this model is considerably reduced.

**Keywords:** dispersive ground; grounding grid; lightning stroke.

### Introduction

Grounding systems such as horizontal electrodes, vertical rods, and grounding grids play an important role in discharging lightning current striking telecommunication towers into ground. To this aim, they are should be correctly designed. Design of such grounding devices is dependent on accurately computing transient voltage of them. Hence, a number of approaches either in time domain [3-5] or frequency domain [6-9] have been proposed. Time domain methods are suitable

for transient analysis in only-ionized ground, whereas frequency domain ones are suitable for inclusion of dispersion of ground. For instance, in transient analyses of grounding systems buried in dispersive ground, K. Sheshyekani et al and S. Visacro et al used finite element method (FEM) [7], hybrid electromagnetics (HEM) [8, 9] respectively. To consider both two effects, the hybrid approach based on combining method of moments (MoM) [10] and vector fitting [11] has been used by J. He et al [12].

All of the above numerical methods solve Maxwell's equations in somehow. These approaches, however, suffers from time consuming and complex computations which from engineering point of view are not advantageous. In contrast with accurate models, approximate models have been proposed. These include transmission line modeling method in one dimension (TLM-1-D) [13], multi-conductor transmission line model (MTL) [14]. In one hand, since TLM-1-D is a time domain approximate model, ground ionization can be easily incorporated, however, soil dispersion cannot be treated. On the other hand, the MTL as a frequency domain approximate model can be used in transient analyses of grounding grids buried in dispersive grounds. Validity of the MTL in transient analyses of grounding systems buried in grounds of constant electrical parameters was only investigated in [14]. In this study, capability of the MTL both in analysis and in design (effective area) of grounding grids buried in dispersive grounds is investigated. The simulation results show that the MTL is suitable in practical applications. This modeling approach is completely explained in the next section.

This paper is organized as follows. In section II, modeling principles of the MTL is explained. Section III is focused on analysis

and design of grounding grids buried in dispersive ground. Finally concluding remarks are given in section IV.

### Modeling Principles

$$-\frac{d^2}{dx^2} V = ZYV = PV \quad (1)$$

$$-\frac{d^2}{dx^2} I = YZI = P_t V \quad (2)$$

where  $Z$  and  $Y$  represent, respectively, the series impedance and parallel admittance per unit length,  $I$  and  $V$  are, respectively, the phasor of current and voltage,  $P = ZY$ ,  $P_t = YZ$

$$I_s = Y_0 \coth(\Psi l) V_s - Y_0 \csc h(\Psi l) V_r$$

(3)

$$I_r = -Y_0 \csc h(\Psi l) V_s + Y_0 \coth(\Psi l) V_r$$

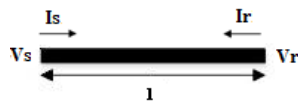
(4)

Rewriting (3) and (4) in matrix form, we have

$$\begin{bmatrix} I_s \\ I_r \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_s \\ V_r \end{bmatrix} \quad (5)$$

where  $A = D = Y_0 \coth(\Psi l)$  and  $B = C = -Y_0 \csc h(\Psi l)$ .  $V_s$  and  $I_s$  represent, respectively, the voltage and current at the sending end of the line, and  $V_r$  and  $I_r$  are, respectively, the voltage and current at the receiving end of the line. Also,  $\Psi$  and  $l$  denotes the propagation constant and length of transmission line respectively. Note that

(a)



### MTL-Based Formulation

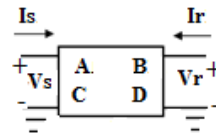
The main idea of the MTL is based on solving the following set of equations describing the propagation phenomenon in transmission lines:

and  $x$  is the variable of length.

Applying a linear transformation in order to diagonalize  $P$  and  $P_t$ , solutions to (1) and (2) can be expressed as follows

$V_s$  and  $V_r$  are sending and receiving voltages with the respect to a point at infinite. Using (5), relation between sending and receiving currents and voltages for a conductor of length  $l$  can be represented as a two-port network as shown in figure 1.

(b)



**Figure (1) (a): Single conductor with sending and receiving voltages and currents. (b): Its representation as a two-port network.**

Now consider a grounding grid of  $1 \times 1$  mesh (with four conductors) as shown in figure 2(a). If each conductor in this figure is

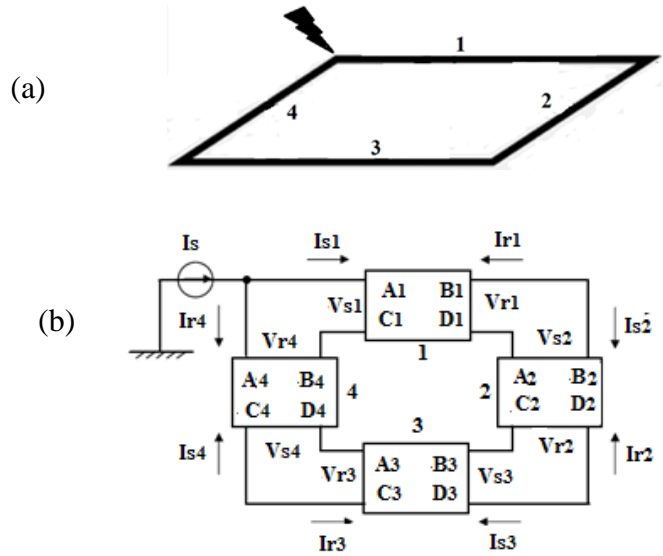
$$\begin{bmatrix} I_{s1} \\ I_{r1} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_{s1} \\ V_{r1} \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} I_{s2} \\ I_{r2} \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{s2} \\ V_{r2} \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} I_{s3} \\ I_{r3} \end{bmatrix} = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \begin{bmatrix} V_{s3} \\ V_{r3} \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} I_{s4} \\ I_{r4} \end{bmatrix} = \begin{bmatrix} A_4 & B_4 \\ C_4 & D_4 \end{bmatrix} \begin{bmatrix} V_{s4} \\ V_{r4} \end{bmatrix} \quad (9)$$

represented as two-port network, figure 2(b) is achieved in which



**Figure 2 (a): Grounding grid consisting of single  $1 \times 1$  mesh, (b): modelling mesh via two-port networks.**

In figure 2(b),  $I_s$  is a current source representing the lightning stroke in figure 3(a). As seen in figure 3(b),  $V_{ri} = V_{s(i+1)}$

$I_{ri} + I_{s(i+1)} = 0, i=1,2,3$  and  $I_{r4} + I_{s1} = I_s$ . Now,

$$\begin{bmatrix} I_s \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A_1 + D_4 & B_1 & 0 & C_4 \\ C_1 & D_1 + A_2 & B_2 & 0 \\ 0 & C_2 & D_2 + A_3 & B_3 \\ B_4 & 0 & C_3 & D_3 + A_4 \end{bmatrix} \begin{bmatrix} V_{s1} \\ V_{r1} \\ V_{r2} \\ V_{r3} \end{bmatrix} \quad (10)$$

or

$$\begin{bmatrix} V_{s1} \\ V_{r1} \\ V_{r2} \\ V_{r3} \end{bmatrix} = \begin{bmatrix} A_1 + D_4 & B_1 & 0 & C_4 \\ C_1 & D_1 + A_2 & B_2 & 0 \\ 0 & C_2 & D_2 + A_3 & B_3 \\ B_4 & 0 & C_3 & D_3 + A_4 \end{bmatrix}^{-1} \begin{bmatrix} I_s \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

Eq. (11) should be solved at each frequency from spectral content of lightning current. Finally  $v_{s1}(t)$ , i.e., time representation of

$$v_{s1}(t) = \sum_{m=1}^N V_{s1,m} \cos(2\pi f_m t + \phi_m) \quad (12)$$

applying these relations on (6) to (9), and combining them the following equation is achieved.

sending voltage at the junction point of the first and fourth conductors is easily computed as bellow

Where  $V_{s1,m}$ ,  $\phi_m$  are respectively magnitude and phase of  $V_{s1}$  at frequency  $f_m$ .

In this study, the lightning current is a slow waveform (first stroke current) and represented as bellow

Lightning Current

$$i(t) = \frac{I_{01}}{\eta} \frac{(t/\tau_1)^n}{1 + (t/\tau_2)^n} e^{(-t/\tau_2)} \quad (13)$$

Where

$$\eta = e^{-(\tau_1/\tau_2)(n(\tau_2/\tau_1))^{1/n}} \quad (14)$$

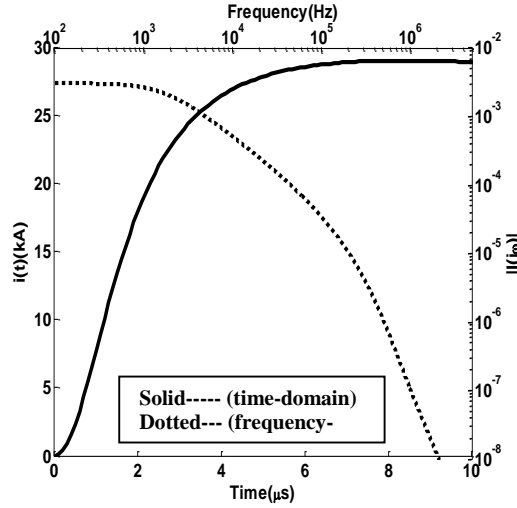
All parameters in (12) are listed in table 1.

**Table 1-Parameters of first stroke current. Adapted from [15].**

$I_0(\text{kA})$	$\tau_1(\mu\text{s})$	$\tau_2(\mu\text{s})$	$n$
28	1.8	95	2

Figure 3 shows time-domain and frequency-domain representations of the first stroke current. In this figure, the right axis represents the magnitude of lightning current

in the frequency domain ( $I_s$ ) with spectral content begins from dc to a few MHz, whereas the left one represents the first stroke current in time domain.



**Figure 3 Representation of first stroke current in the frequency domain (right axis) and in time domain (left axis).**

Dispersive Ground

Since the focus in this study is on validity of the MTL in dispersive grounds, the frequency variation of electrical parameters of such

grounds in this sub-section is represented. There are a few models representing such phenomenon [6], but without losing generality, the proposed model by S. Visacro et al [16] is used as bellow

$$\sigma(f) = \sigma_0 \left( 1 + \left( 1.2 \cdot 10^{-6} \cdot \sigma_0^{-0.73} \right) (f - 100)^{0.65} \right) \quad (15)$$

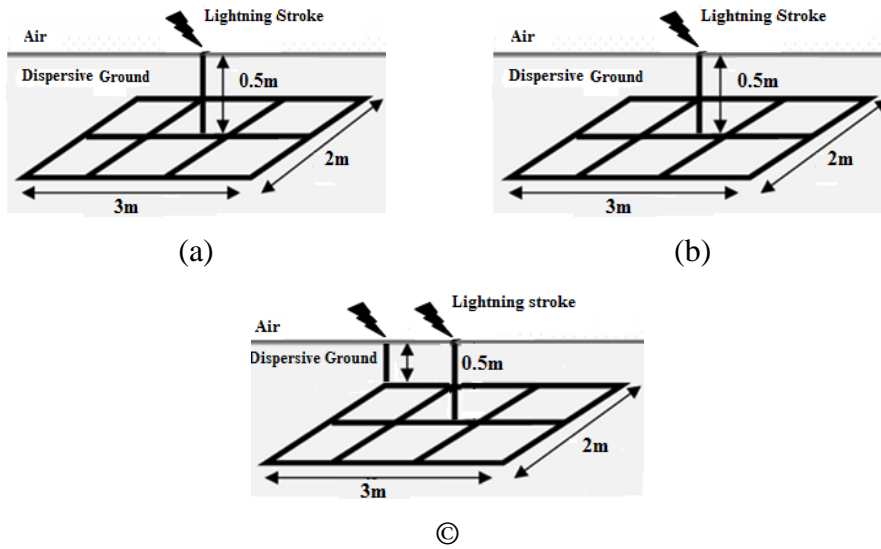
$$\epsilon_r(f) = \begin{cases} 192.2 & f \leq 10\text{kHz} \\ 1.3 + 7.6 \cdot 10^3 \cdot f^{-0.4} & f \geq 10\text{kHz} \end{cases} \quad (16)$$

Where  $\sigma_0$  is low-frequency conductivity of the ground.

### Numerical simulation and Verification Grounding Potential Rise (GPR)

In this section with the aim of validity, the MTL is applied on a grounding grid from [7] and shown in figure 4. As seen, the grid is an equally spaced  $2\text{m} \times 3\text{m}$  square and buried in depth of  $0.5\text{m}$ . Each mesh within the grid is a

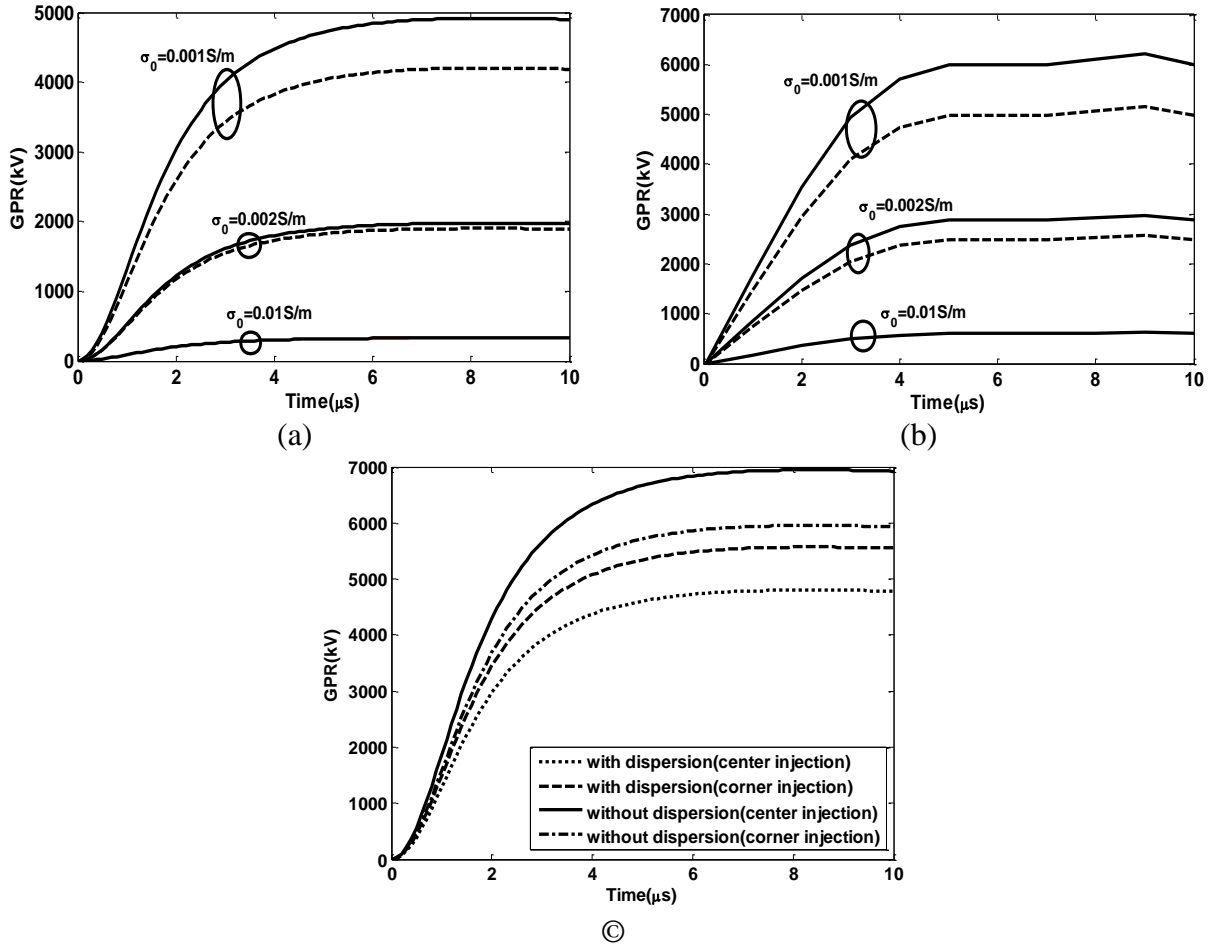
$1\text{m} \times 1\text{m}$  square. In [7], such a grounding grid is analyzed based on finite element method (FEM). In this article, without losing generality, first stroke current is injected at center, corner of the grid separately or simultaneously. Simulation results for grounding potential rise (GPR) via the MTL is shown in figure 5.



**Figure (4) Schematic diagram of grounding grid buried in dispersive ground. (a): Center injection, (b): Corner injection, and (c): simultaneous injection at center and corner of the grid (two-port grid). Adapted from [7].**

In this figure, dashed and solid lines denote with and without considering dispersion of the ground respectively. It is seen that good agreement with the individual ones in [7] is achieved (see figure 12(a) in [7]).

In [7], the analysis was only carried out on the center injection. To further demonstrate the capability of the MTL, such a grounding grid is also corner-injected by the first stroke current. The simulation results are shown in figure 5.



**Figure (5) Grounding potential rise (GPR) for the  $2\text{m} \times 3\text{m}$  grid for (a): center injection, (b): corner injection, and (c): center and corner injection simultaneously (two-port grid with  $\sigma_0 = 0.001\text{S/m}$ ).**

As it is seen, corner injection results are greater than the individual ones for center injection which is in consistent with [17]. Finally, when the grid is injected at its center and corner simultaneously (two-port grid), GPRs at the two ports for  $\sigma_0 = 0.001\text{S/m}$  are computed and shown in figure 5©.

#### Effective Area

In this section, validity of the MTL in designing grounding grids buried in dispersive grounds is investigated. With reference to [18], the effective area ( $A_{\text{eff}}$ ) is defined as an area beyond which the grounding grid is ineffective, i.e., an area within which the lightning current is completely discharged. This phenomenon is illustrated in figure 9 which is adopted from [19]. In this figure, the concept of effective length/area is illustrated. To compute these

quantities, impulse impedance should be first defined and computed. Impulse impedance ( $Z_p$ ) is defined as the ratio of peak values of GPR and lightning current [18]. Then, the effective length ( $L_{\text{eff}}$ ) is defined as the threshold value for the grid-side length since then impulse impedance becomes constant. Finally, as illustrated in figure 9, for corner injection,  $A_{\text{eff}}$  corresponds to the half value of  $(L_{\text{eff}})^2$  whereas for center injection it is equal to two times  $(L_{\text{eff}})^2$ . Consequently the effective area for center injection is four times that of corner injection as illustrated in figure 6. Gray circles in figure 6 denote effective areas. Note that  $L_{\text{eff}}$  for center and corner injection is the same.

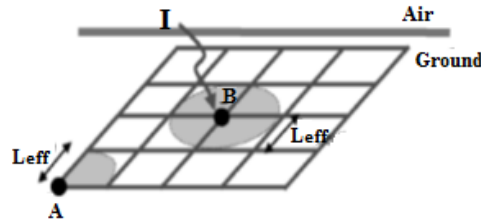
Figure 7 shows  $Z_p$  for three values of  $\sigma_0$ . From this figure,  $L_{\text{eff}}$  is easily computed and

shown as solid circles for three values of  $\sigma_0$ . Now  $A_{\text{eff}}$  is computed via MTL and compared with the HEM-based extracted formulae in [9]. These formulae expressed as bellow are only for grounding grids under first stroke current

$$A_{\text{eff}}|_{\text{corner}} = 1/2(L_{\text{eff}})^2 \quad (16)$$

$$A_{\text{eff}}|_{\text{center}} = 2(L_{\text{eff}})^2 \quad (17)$$

Where



**Figure (6) Illustration of effective areas (grey circles) for center (B) and corner (A) injections of a typical grounding grid. Adapted from [19].**

Table 2 compares the computed effective length for the grounding grid under consideration using MTL and HEM. As seen in this table, good agreement between the proposed model and accurate model is achieved. The slight difference between the

$$L_{\text{eff}} = I_{\text{C0-1st}} / \alpha_{\text{1st}}$$

(18)

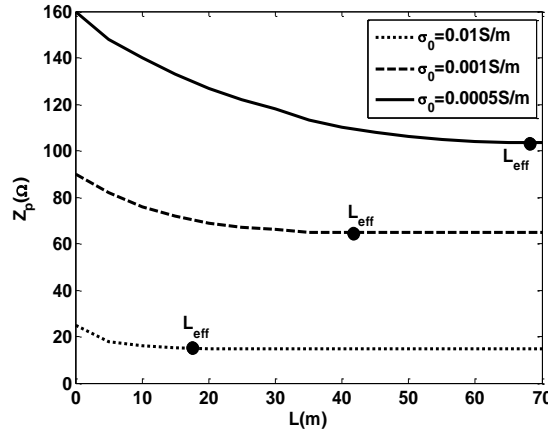
$$I_{\text{C0-1st}} = -0.00086 \times (\sigma_0)^{-0.686} + 0.992$$

$$\alpha_{\text{1st}} = (1/k) [0.002924 + \exp(-0.8935 \times \sigma_0^{-0.2129})]$$

(19)

Equations (16) and (17) represent effective area for corner and center injections respectively. Also in (19),  $k$  is a constant equal to 1 and 2 for corner and center injection respectively.

two models may be due to truncation error of the lightning current in the frequency domain. Finally the effective area for different injections are computed and listed in table 3.



**Figure (7) Impulse impedance for the grounding grid versus grid-side length for three values of  $\sigma_0$  (for center and corner injections).**

**Table 2- Comparing Effective lenth ( $L_{\text{eff}}$ ) in (m) for corner and center-injected grounding grid via MTL and HEM.**

0.01(S/m)		0.001(S/m)		0.0005(S/m)	
MTL	17	MTL	42	MTL	68
HEM	15	HEM	45	HEM	70



**Table 3- Comparing effective area ( $A_{eff}$ ) in  $m^2$  based on the MTL for center and corner-injected grounding grid.**

$\sigma_0$ (S/m)			
<b>Injection</b>	<b>0.01</b>	<b>0.001</b>	<b>0.0005</b>
<b>Corner</b>	144.5	882	2312
<b>Center</b>	578	3528	9248

## Conclusion

In this study, the MTL-based approach was used for transient analysis and design of grounding systems. The simulation results show that firstly in comparison with accurate approaches such as FEM and HEM, it is in good agreement, secondly computation time is vanishingly short.

## References

- [1] J. He, "Progress in Lightning Impulse Characteristics of Grounding Electrodes With Soil Ionization," IEEE Transactions on Industry Applications, vol. 51, no. 6, pp. 4924-4933, November/December 2015.
- [2] S. Visacro, "What engineers in industry should know about the response of grounding electrodes subjected to lightning currents," IEEE Transactions on Industry Applications, vol. 51, no. 6, pp. 4943-4951, November/December 2015.
- [3] C. M. Portela, M. C. Tavaras, and P. Filho, "Accurate representation off soil behavior for transient studies," IEE Proceedings of Generation, Transmission and Distribution, vol. 150, no. 6, November 2003.
- [4] G. Ala, P. L. Buccheri, P. Romano, and F. Viola, "Finite difference time domain simulation of earth electrodes soil ionization under lightning surge conditions," IET Science, Measurement Technology, vol. 2, no. 3, pp. 134-135, May 2008.
- [5] Z. Feng, X. Wen, X. Tong, H. Lu, L. Lan, and P. Xing, "Impulse Characteristics of Tower Grounding Devices Considering Soil Ionization by the Time-Domain Difference Method," IEEE Transactions on Power Delivery, vol. 30, no. 4, pp. 1906-1913, August 2015.
- [6] D. Cavka, N. Mora, and F. Rashidi, "A Comparison of Frequency-Dependence Soil Models: Application to the Analysis of Grounding systems," IEEE Transactions on Electromagnetic Compatibility, vol. 56, no. 1, pp. 177-187, April 2013.
- [7] Majed Akbari, K. Sheshyekani, Mohammad Reza Alemi, "The Effect of Frequency Dependence of Soil Electrical Parameters on the Lightning Performance of Grounding Systems," IEEE Transactions on Electromagnetic Compatibility, vol. 55, no. 4, pp. 739-746, April 2013.
- [8] Rafael Alipio, and S. Visacro, "Impulse Efficiency of Grounding Electrodes: Effect of Frequency-dependence Soil Parameters," IEEE Transactions on Power Delivery, vol. 29, no. 2, pp. 716-723, April 2014.
- [9] Rafael Alipio, and Silverio Visacro, "Impulse Efficiency of Grounding Grids: Effect of Frequency-dependence Soil Parameters," IEEE Transaction on Power Delivery, vol. 29, no. 2, pp. 102-108, August 2015.
- [10] L. Grcev and F. Dawalibi, "An electromagnetic model for transients in grounding systems," IEEE Trans. Power Del., vol. 5, no. 4, pp. 1773-1781, October 1990.
- [11] Bjorn Gustavsen, and Adam Semlyen, "Rational Approximation of Frequency Domain Responses by Vector Fitting," IEEE Transaction on Power Delivery, Vol. 14, No. 3, 1999., No. 2, pp. 517-524, 1999.
- [12] J. Wu, B. Zhang, J. He, and R. Zeng, "A Comprehensive Approach for Transient Performance of Grounding System in the Time Domain," IEEE Transactions on Electromagnetic Compatibility, vol. 57, no. 2, pp. 250-256, April 2015.
- [13] R. S. Bretas, Guilherme A. D. Dias, Marcos Tello, Dave W. P. Thomas, and Christos Christopoulos, "The Transmission Line Modeling Method to Represent the Soil Ionization Phenomenon in Grounding Systems," IEEE Transactions on Magnetics, vol. 50, no. 2, pp. 1163-1171, February 2014.
- [14] A. Jardines, J. L. Guardado, J. Torres, J. J. Chavez, M. Hernandez, "A Multiconductor Transmission Line Model for Grounding Grid," Electrical Power and Energy Systems, vol. 60, pp. 24-33, April 2014.
- [15] F. Rachidi, W. Janischewskyj, A. M. Hussein, C. A. Nucci, S. Guerrieri, B. Kordi, and J.-S. Chang, "Current and electromagnetic field associated with lightning-return strokes to tall towers," IEEE Trans. Electromagn. Compat, vol. 43, no. 3, pp. 356-367, Aug. 2001.
- [16] S. Visacro and Rafael Alipio, "Frequency Dependence of Soil Parameters: Experimental Results, Predicting Formula and Influence on the Lightning Response of Grounding Electrodes," IEEE Transactions on Electromagnetic Compatibility, vol. 27, no. 2, pp. 927-935, April 2012.
- [17] L. Crcev, "Lightning Surge efficiency of Grounding Grids," IEEE Transaction on Power Delivery, vol. 26, no. 3, pp. 1692-1699, April 2011.
- [18] Yaqing Liu, Nelson Theethayi, and Rajeev Thottappillil, "Investigating the Validity of Existing Definitions and



Empirical Equations of Effective Length/Area of Grounding Wire/Grid for Transient studies”, Journal of Electrostatic, vol. 65, pp. 329-335, 2007.

- [19] S. Visacro, “A Comprehensive Approach to the Grounding Response to Lightning Currents”, IEEE Transaction on Power Delivery, vol. 22, no. 1, April 2007.