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Simultaneous Wireless Information and Power Transfer Optimization with Imperfect Channel State Information

Saeed Allahzadeh
Received M.S. degree in communication engineering, Electrical Eng. Dept. , Imam Reza International University, Mashhad, Iran
S.Allahzadeh@imamreza.ac.ir
Ebrahim Daneshifar
Post Doc in communication, Assistant professor, Biomedical Eng. Dept. , Imam Reza International University, Mashhad, Iran.
Ebrahim@imamreza.ac.ir

Abstract
Simultaneous Wireless Information and Power Transfer (SWIPT) systems are vital to implement wireless charging in contemporary Broadcast Channel (BC) systems. We would design a multi-user multi-antenna BC by simultaneously maximizing the sum of harvested energy and minimizing the sum Mean Square Error (MSE) for symbol detection at each receiver under imperfect Channel State Information (CSI) condition. We will find the minimum sum of harvested energy and exploit the Semidefinite Relaxation (SDR) method to find the worst case of the sum MSE. This multi-objective problem can be recast as a Difference of Convex (DC) bilinear problem. The derived problem is solved using the Alternating Convex Search (ACS) method and the Penalty Convex-Concave Program (PCCP) procedure. Simulation results are employed to address the benefits of the proposed algorithm.

Keywords : Simultaneous Wireless Information and Power Transfer (SWIPT) systems, Difference of Convex (DC) problem, Penalty Convex-Concave Program (PCCP) procedure, Robust optimization

Introduction
One of the major challenges in Internet of Things (IoT) Network is to provide power to a large number of sensors. As you know, wireless signals carry both energy and information at the same time, therefore SWIPT (Simultaneous Wireless Information and Power Transfer) systems use this feature of wireless signals to transfer power and information concurrently. Hence SWIPT technology can be a promising solution to supply power for rising sensors in 5G infrastructure. In the SWIPT network, because the need to replace or charge the battery is eliminated, the network will never be out of service, so the SWIPT will increase the efficiency of the network.
Because there are no standards and products in SWIPT yet, therefore must be much research and studying in this field. We have to deal with different aspects of this technology, such as information theory, circuit theory, communication theory, optimization, ⋯ . We analyze a SWIPT system from the perspective of optimization [1].
As you know increasing information rate reduces harvested energy and vice versa in SWIPT system [2]. Thus, a compromise is needed to determine the parameters of the system so that both the criteria of information rate and harvested energy in the system are maximized. In most papers, only one criterion is optimized and the other one criterion considered as a constraint in the problem [3]-[4]. Unlike most research papers, in [5] and [6] both criteria, harvested energy and information rate, are optimized simultaneously.
In designing wireless communication systems, the Channel State Information (CSI) is a parameter with no control over it. Therefore, with imperfect CSI, optimizing a wireless communication system is a hard

work. Usually, the precoders and equalizers of a wireless communication system are designed in such a way that if there are any fluctuations around nominal values, still some optimality conditions are satisfied [7]. In [8], there are one Road-Side Unit (RSU) with N_t transmit antenna and one single-antenna legitimate vehicle (LV) and k Eavesdropping vehicles (EVs) . The aim of this paper is to maximize the Secure Energy-Efficient (SEE), which is defined as the ratio of the achievable secrecy rate to the total power consumption of the system, with constraints of secrecy rate of the LV and the amount of the harvested energy by LV under imperfect CSI condition. By exploiting the maximum ratio transmission (MRT) scheme and the norm-bounded matrix theory, the authors solve this non-convex problem. In [9], a two-user cooperative non-orthogonal multiple access (NOMA) transmission scheme is considered, by which the transmitter is equipped with N antennas and the two users, cell-center or near user and cell-edge or far user are equipped with a single antenna. . The aim of the system is to maximize the data rate of the near user while satisfying the data rate requirement of the far user. In [10], a secure relaying communication system, which consists of one single-antenna source and eavesdropper and a full-duplex (FD) legitimate destination with a dual separate antenna and a multi-antenna relay. The CSI between eavesdropper and relay, and eavesdropper and destination are imperfect. The relay transmit data to the destination when harvest energy from the source. the FD destination also acts as a jammer to cooperatively transmit artificial noise (AN) to degrade the received signal interference-plus-noise ratio (SINR) at the eavesdropper. The aim of this paper is to find precoding matrix and power splitter ratio at relay and AN power by maximizing the secrecy rate under the constraints of energy harvesting (EH) requirement at the relay and AN transmit power at the destination. This problem is a non-convex problem which authors solve it by using Semidefinite Relaxation (SDR) and successive convex optimization methods.

In the current paper, we want to design a Multi-User Multi-Input Multi-output (MU-MIMO) SWIPT system with k receivers each of which has two parts, information decoder (ID) and energy harvsting (EH) node. Every ID part is equipped with a decoder. Corresponding with each receiver, there is a precoder matrix in the transmitter.

In our proposed method, we want to jointly optimize the precoders and decoders by simultaneously maximizing the sum of all energy harvested in all receivers, and minimizing the sum Mean-Square Error (MSE) of the symbol detection in them, with a constraint on the transmit power of the BS under imperfect CSI condition, the CSI of all links is uncertain and we would resort to Strictly Bounded Robust- Semidefinite Relaxation (SBR-SDR) [11],[12]. As it can be seen, this problem can be cast as global multi-objective program which is a non-convex problem, i.e., a Difference of Convex (DC) problem. To solve DC problem we use a Penalty Convex-Concave Program (PCCP) procedure [13],[14] in which a first-order approximation of the convex function is used.

System Model

Let's assume a MU-MIMO SWIPT system with one BS, and k users in a broadcasting configuration, i.e., one transmitter and k receivers. The BS is equipped with n antennas and each of the receivers has m antennas. Vector of $\mathbf{s}_i \in \mathbb{C}^{t_i \times 1}, i = 1, \cdots, k$ is transmitted to the ith receiver by BS, in which, t_i is the number of symbols of the ith receiver. We suppose that the distribution of these symbols are complex normal with zero-mean and variance of one, i.e., $\mathbf{s}_i \sim \mathrm{CN}(\mathbf{0}, \mathbf{I}_{t_i})$. The noise at each receiver is also assumed to have a complex normal distribution, i.e., $\mathbf{n}_i \in \mathbb{C}^{m \times 1} \sim \mathrm{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_\mathbf{m})$. At the transmitter side, each vector of symbols is precoded using a matrix $\mathbf{A}_i \in \mathbb{C}^{n \times t_i}$ and would be equalized at the receiver side using a matrix $\mathbf{B}_i \in \mathbb{C}^{t_i \times m}$. At the end, the output vector at each of the receivers is denoted by $\mathbf{z_i} \in \mathbb{C}^{t_i \times 1}$. In addition, the power splitter splits the received power, P_r, into two parts controlled using a parameter, α $(0 \leq \alpha \leq 1)$. αP_r portion of it is sent to the ID module while the rest of it, $(1 - \alpha)P_r$, is sent to the EH module. After conversion, this energy is saved in a battery (BAT) that is used in uplink the mode. We also assume that the conversion efficiency of the EH module is $0 \leq \eta \leq 1$.

It should also be noted that the channel matrices of $\mathbf{H}_i \in \mathbb{C}^{m\times n}$, between the BS and the ith user, are assumed to be imperfect and we want to design of the aforementioned system by solving a robust optimization problem with SBR-SDR method. The message at the BS is denoted by $\mathbf{x} \in^{n\times 1}$. It can be written as $\mathbf{x} = \sum_{i=1}^{k} \mathbf{A}_i\mathbf{s}_i$.

The received signals of each reciever in the ID module can be written as:

$$\mathbf{y}_{i,ID} = \sqrt{\alpha}\,(\mathbf{H}_i\mathbf{x} + \mathbf{n}_i) = \sqrt{\alpha}\left(\mathbf{H}_i\sum_{j=1}^{k}\mathbf{A}_j\mathbf{s}_j + \mathbf{n}_i\right)$$
$$= \sqrt{\alpha}\,\mathbf{H}_i\mathbf{A}_i\mathbf{s}_i + \sqrt{\alpha}\,\sum_{j=1\neq i}^{k}\mathbf{H}_i\mathbf{A}_j\mathbf{s}_j + \sqrt{\alpha}\mathbf{n}_i \qquad (1)$$

while the received signal of EH module would be

$$\mathbf{y}_{i,EH} = \sqrt{1-\alpha}\left(\sum_{j=1}^{k}\mathbf{H}_i\mathbf{A}_j\mathbf{s}_j + \mathbf{n}_i\right). \qquad (2)$$

At each receiver, the ID module would decode its received signal using B_i matrices and thus the output signal of each ID module would be

$$\mathbf{z}_i = \mathbf{B}_i\mathbf{y}_{i,ID} = \sqrt{\alpha}\,\mathbf{B}_i\mathbf{H}_i\mathbf{A}_i\mathbf{s}_i + \sqrt{\alpha}\,\sum_{j=1\neq i}^{k}\mathbf{B}_i\mathbf{H}_i\mathbf{A}_j\mathbf{s}_j + \sqrt{\alpha}\mathbf{B}_i\mathbf{n}_i \qquad (3)$$

The Proposed Algorithm with Imperfect CSI

We want to find the best $\mathbf{A}_i$ and $\mathbf{B}_i$, $i = 1, \cdots, k$ matrices, such that the sum of harvested energy in their EH module is maximized while the sum MSE over all receivers is simultaneously minimized. It means that we want to solve both P1 and P2 problems at the same time:

$$\text{(P1)} \quad \min_{\{\mathbf{A}_i,\mathbf{B}_i\}_{i=1}^{k}} \sum_{i=1}^{k}\mathrm{MSE}_i \qquad\qquad \text{(P2)} \quad \max_{\{\mathbf{A}_i\}_{i=1}^{k}} \sum_{i=1}^{k} Q_i$$

$$\text{s.t.} \quad \mathrm{TxP} \leq P \qquad\qquad \text{s.t.} \quad \mathrm{TxP} \leq P$$
$$\mathbf{H}_i \in \mathcal{H}_i \qquad\qquad \mathbf{H}_i \in \mathcal{H}_i \qquad (4)$$

Where

$$Q_i = \eta\,\mathbb{E}[\|\,\mathbf{y}_{i,EH}\,\|^2] = \eta(1-\alpha)\left[\sum_{j=1}^{k}\|\,\mathbf{H}_i\mathbf{A}_j\,\|_F^2 + m\sigma_n^2\right]$$

is the harvested energy[1] in each node

$$\mathrm{TxP} = \mathbb{E}[\|\,\mathbf{x}\,\|^2] = \sum_{i=1}^{k}\|\,\mathrm{vec}(\mathbf{A}_i)\,\|^2$$

is the transmit power of the BS and it is limited by P, and

$$\mathrm{MSE}_i = \mathbb{E}[\|\,\mathbf{z}_i - \mathbf{s}_i\,\|^2] = \|\,\sqrt{\alpha}\mathbf{B}_i\mathbf{H}_i\mathbf{A}_i - \mathbf{I}_{t_i}\,\|_F^2 + \alpha\sigma_n^2\,\|\,\mathbf{B}_i\,\|_F^2 + \alpha\sum_{j=1\neq i}^{k}\|\,\mathbf{B}_i\mathbf{H}_i\mathbf{A}_j\,\|_F^2$$

2- For convenience, in the sequel of the paper the two terms “energy” and “power” may be used interchangeably by assuming the symbol period to be equal to one.

is the MSE of the ith link and we assume every link is modeled with a Norm-Bounded Error (NBE), i.e.,

$$\mathbf{H_i} \in \boldsymbol{\mathcal{H}}_\mathbf{i} = \left\{ \widehat{\mathbf{H}}_\mathrm{i} + \boldsymbol{\Delta}_\mathrm{i} \mid \ \| \boldsymbol{\Delta}_\mathrm{i} \|_\mathrm{F} \leq \ \delta_\mathrm{i} \ \right\} \qquad (5)$$

in which $\widehat{\mathbf{H}}_\mathrm{i}$ is the nominal value of the channel, $\boldsymbol{\Delta}_\mathrm{i}$ is the CSI uncertainty matrix and finally δ_i is the uncertainty bound.
in this optimization problem. The problem, P1 is a bi-convex problem, i.e., it is not simultaneously convex with respect to both arguments, but if one parameter, for example, $\mathbf{A}_\mathrm{i}\forall\mathrm{i}$, is fixed, the problem would be a convex one with respect to the second parameter. Problem P1 is studied in [12], and the authors use Alternating Convex Search (ACS) [15] to solve it. To solve both P1 and P2 problems simultaneously, which is a multi-objective non-convex problem, we use vector optimization for finding pareto optimal (or optimal) points [16]. Therefore the epigraph form of P1 and P2 can be written as (6) which is a convex-concave problem:

$$\min_{\mathbf{A},\mathbf{B},\{\tau_\mathrm{i}\,,\gamma_\mathrm{i}\}_{\mathrm{i}=1}^{\mathrm{k}}} \left\{ \sum_{\mathrm{i}=1}^{\mathrm{k}} \tau_\mathrm{i} - \gamma_\mathrm{i} \left| \begin{array}{l} \mathrm{TxP} \leq \mathrm{P}\,, \mathrm{h_i}(\mathbf{A}) \geq \gamma_\mathrm{i}\,, \ \ \forall\, \mathrm{i} \\ \| \boldsymbol{\mu}_\mathrm{i} \|^2 \leq \tau_\mathrm{i}\,, \mathbf{H_i} \in \mathcal{H}_\mathrm{i}\,, \ \forall \mathrm{i} \end{array} \right. \right\} \qquad (6)$$

in which

$$\mathbf{A} = [\mathbf{A_1}, \dots, \mathbf{A_k}]\,, \mathbf{B} = [\mathbf{B_1}, \dots, \mathbf{B_k}]\,, \quad \mathrm{h_i}(\mathbf{A}) = \eta(1-\alpha)\left[\sum_{\mathrm{j}=1}^{\mathrm{k}} \mathrm{tr}\left(\mathbf{H_i A_j A_j^H H_i^H}\right) + \mathrm{m}\sigma_\mathrm{n}^2\right] \qquad (7)$$

$$\boldsymbol{\mu}_\mathbf{i} = \begin{bmatrix} \sqrt{\alpha}\left(\mathbf{A_i^T} \otimes \mathbf{B_i}\right)\mathrm{vec}(\mathbf{H_i}) - \mathrm{vec}(\mathbf{I}_{\mathrm{t_i}}) \\ \sqrt{\alpha}\ \mathrm{MAT}[\{\left(\mathbf{A_j^T} \otimes \mathbf{B_i}\right)\mathrm{vec}(\mathbf{H_i})\}_{\mathrm{j}=1\neq\mathrm{i}}^{\mathrm{k}}] \\ \sqrt{\alpha}\sigma_\mathrm{n}\mathrm{vec}(\mathbf{B_i}) \end{bmatrix} \qquad (8)$$

The (6) is a bi-linear DC problem which we would tackle it using PCCP with bi-convex semi-infinite constraints. Because of bi-linearity we would use ACS and due to having semi-infinite constraints we should find the worst channel realization in which $\mathrm{h_i}$ is minimized (SBR) and $\| \boldsymbol{\mu}_\mathrm{i} \|^2$ is maximized (SDR).

SBR Method
The (6) can be recasted as

$$\min_{\mathbf{A},\mathbf{B},\{\tau_\mathrm{i}\,,\gamma_\mathrm{i}\}_{\mathrm{i}=1}^{\mathrm{k}}} \sum_{\mathrm{i}=1}^{\mathrm{k}} \tau_\mathrm{i} - \gamma_\mathrm{i} \qquad (9a)$$

$$\mathrm{s.t.} \qquad \mathrm{TxP} \leq \mathrm{P} \qquad (9b)$$

$$\left[\min_{\mathbf{H_i}\in\mathcal{H}_\mathrm{i}} \mathrm{h_i}(\mathbf{A})\right] \geq \gamma_\mathrm{i} \quad \forall \mathrm{i} \qquad (9c)$$

$$\| \boldsymbol{\mu}_\mathrm{i} \|^2 \leq \tau_\mathrm{i} \qquad \forall \mathrm{i} \qquad (9d)$$

$$\| \boldsymbol{\Delta}_\mathrm{i} \|_\mathrm{F} \leq \ \delta_\mathrm{i} \qquad \forall \mathrm{i} \qquad (9e)$$

To solve the inner problem of (9c) we start using (5)

$$\boldsymbol{H_i^H H_i} = \left(\widehat{\boldsymbol{H}}_i + \boldsymbol{\Delta}_i\right)^H \left(\widehat{\boldsymbol{H}}_i + \boldsymbol{\Delta}_i\right) = \widehat{\boldsymbol{H}}_i^H \widehat{\boldsymbol{H}}_i + \widehat{\boldsymbol{H}}_i^H \boldsymbol{\Delta}_i + \boldsymbol{\Delta}_i^H \widehat{\boldsymbol{H}}_i + \boldsymbol{\Delta}_i^H \boldsymbol{\Delta}_i \quad (10)$$

So, the uncertain set can be rewritten as:

$$\boldsymbol{H_i^H H_i} = \check{\mathrm{H}}_{\boldsymbol{i}} \in \breve{\boldsymbol{\mathcal{H}}}_{\boldsymbol{i}} = \left\{\ddot{\boldsymbol{H}}_{\boldsymbol{i}} + \ddot{\boldsymbol{\Delta}}_i \;\mid\; \| \ddot{\boldsymbol{\Delta}}_i \|_F \leq \boldsymbol{\epsilon_i} \right\} \quad (11)$$

in which

$$\ddot{\boldsymbol{H}}_{\boldsymbol{i}} = \widehat{\boldsymbol{H}}_i^H \widehat{\boldsymbol{H}}_i \,, \ddot{\boldsymbol{\Delta}}_i = \widehat{\boldsymbol{H}}_i^H \boldsymbol{\Delta}_i + \boldsymbol{\Delta}_i^H \widehat{\boldsymbol{H}}_i + \boldsymbol{\Delta}_i^H \boldsymbol{\Delta}_i.$$

So using [11] it is possible to have the following equation

$$\epsilon_i \geq \| \ddot{\boldsymbol{\Delta}}_i \|_F = \| \widehat{\boldsymbol{H}}_i^H \boldsymbol{\Delta}_i + \boldsymbol{\Delta}_i^H \widehat{\boldsymbol{H}}_i + \boldsymbol{\Delta}_i^H \boldsymbol{\Delta}_i \|_F$$

$$\leq \| \widehat{\boldsymbol{H}}_i^H \|_F \| \boldsymbol{\Delta}_i \|_F + \| \boldsymbol{\Delta}_i^H \|_F \| \widehat{\boldsymbol{H}}_i \|_F + \| \boldsymbol{\Delta}_i^H \|_F \| \boldsymbol{\Delta}_i \|_F$$

$$= \delta_i^2 + 2\delta_i \| \widehat{\boldsymbol{H}}_i \|_F \quad (12)$$

Then, it is possible to choose

$$\epsilon_i = \delta_i^2 + 2\delta_i \| \widehat{\boldsymbol{H}}_i \|_F \quad (13)$$

So, using (11) and (7) we have

$$tr\left(\boldsymbol{H_i A_j A_j^H H_i^H}\right) = tr(\boldsymbol{A_j A_j^H H_i^H H_i})$$

$$= tr\left(\boldsymbol{A_j A_j^H}\left(\ddot{\boldsymbol{H}}_{\boldsymbol{i}} + \ddot{\boldsymbol{\Delta}}_i\right)\right). \quad (14)$$

Therefore we can rewrite (9c)

$$\min_{\|\ddot{\boldsymbol{\Delta}}_i\|_F \leq \boldsymbol{\epsilon_i}} \eta(1-\alpha)\left[\sum_{j=1}^{k} tr\left(\boldsymbol{A_j A_j^H}\left(\ddot{\boldsymbol{H}}_{\boldsymbol{i}} + \ddot{\boldsymbol{\Delta}}_i\right)\right) + m\sigma_n^2\right] \quad (15)$$

Equation (15) and then (9) can be simplified by using the following proposition.

Proposition 1: For the terms $tr[(\boldsymbol{B}+\boldsymbol{C})\boldsymbol{A}]$, using a norm-bounded variable $\boldsymbol{C}$, i.e.,$\| \boldsymbol{C} \| \leq \epsilon$, the minimizer and maximizer would , respectively, be

$$\boldsymbol{C}^{min} = -\epsilon \frac{\boldsymbol{A}^H}{\| \boldsymbol{A} \|}, \qquad \boldsymbol{C}^{max} = \epsilon \frac{\boldsymbol{A}^H}{\| \boldsymbol{A} \|}$$

Proof: Proof can be found in [11].
Using the proposition we have:

$$\min_{\|\boldsymbol{C}\| \leq \epsilon} tr[(\boldsymbol{B}+\boldsymbol{C})\boldsymbol{A}] = tr(\boldsymbol{BA}) - \epsilon \| \boldsymbol{A} \| \quad (16)$$

$$\max_{\|\boldsymbol{C}\| \leq \epsilon} tr[(\boldsymbol{B}+\boldsymbol{C})\boldsymbol{A}] = tr(\boldsymbol{BA}) + \epsilon \| \boldsymbol{A} \| \quad (17)$$

and (9) can be recasted as:

$$\min_{\boldsymbol{A},\boldsymbol{B},\{\tau_i,\gamma_i\}_{i=1}^{k}} \sum_{i=1}^{k} \tau_i - \gamma_i \qquad \text{(18a)}$$

$$s.t. \qquad TxP \leq P \qquad \text{(18b)}$$

$$\bar{h}_i(\boldsymbol{A}) \geq \gamma_i \quad \forall i \qquad \text{(18c)}$$

$$\| \boldsymbol{\mu}_i \|^2 \leq \tau_i \quad , \quad \| \boldsymbol{\Delta}_i \|_F \leq \delta_i \quad \forall i \qquad \text{(18d)}$$

in which

$$\bar{h}_i(\boldsymbol{A}) = \eta(1-\alpha)\left[\sum_{j=1}^{k} tr\left(\boldsymbol{A_j}\boldsymbol{A_j^H}\ddot{\boldsymbol{H}}_{\boldsymbol{i}}\right) - \boldsymbol{\epsilon_i} \| \boldsymbol{A_j}\boldsymbol{A_j^H} \|_F + m\sigma_n^2\right] \qquad \text{(19)}$$

SDR Method

The problem of (18) is a semi-infinite problem. It is possible to use the Schur complement lemma and recast (18d) as a semidifinite program (SDR) [12]. Using the Schur complement lemma (18d) is rewritten as an Linear Matrix Inequality (LMI):

$$\begin{bmatrix} \tau_i & \boldsymbol{\mu}_i^H \\ \boldsymbol{\mu}_i & \boldsymbol{I} \end{bmatrix} \succcurlyeq 0 \, , \| \boldsymbol{\Delta}_i \|_F \leq \delta_i \quad \forall i. \qquad \text{(20)}$$

To simplify (20), we use the following lemma which is due to Nemirovski [17].

Lemma 1: Given matrices $\boldsymbol{W}, \boldsymbol{Y}, \boldsymbol{Z}$ with $\boldsymbol{Z} = \boldsymbol{Z^H}$ the semi-infinite LMI of the form of $\boldsymbol{Z} \geq \boldsymbol{W^H X Y} + \boldsymbol{Y^H X^H W}, \ \forall \boldsymbol{X}: \| \boldsymbol{X} \| \leq \rho$ holds if and only if $\exists \lambda \geq 0$ such that $\begin{bmatrix} \boldsymbol{Z} - \lambda \boldsymbol{Y^H Y} & -\rho \boldsymbol{W}^H \\ -\rho \boldsymbol{W} & \lambda \boldsymbol{I} \end{bmatrix} \succcurlyeq 0$

Proof: Proof can be found in [17].

Using (5), the constraint of (20) can be written as follows:

$$\begin{bmatrix} \tau_i & \widehat{\boldsymbol{\mu}_i^H} \\ \widehat{\boldsymbol{\mu}}_i & \boldsymbol{I} \end{bmatrix} + \begin{bmatrix} 0 & \boldsymbol{\mu}_{\boldsymbol{\Delta}_i}^{\boldsymbol{H}} \\ \boldsymbol{\mu}_{\boldsymbol{\Delta}_i} & \boldsymbol{0} \end{bmatrix} \succcurlyeq 0 \qquad \text{(21)}$$

and

$$\hat{\mu}_i = \begin{bmatrix} \sqrt{\alpha}\left(\boldsymbol{A_i^T} \otimes \boldsymbol{B_i}\right) vec\left(\widehat{\boldsymbol{H}}_i\right) - vec\left(\boldsymbol{I}_{t_i}\right) \\ \sqrt{\alpha}\, MAT\left[\left\{\left(\boldsymbol{A_j^T} \otimes \boldsymbol{B_i}\right) vec\left(\widehat{\boldsymbol{H}}_i\right)\right\}_{j=1\neq i}^{k}\right] \\ \sqrt{\alpha}\sigma_n vec(\boldsymbol{B_i}) \end{bmatrix} \in \mathbb{C}^{y_i \times 1} \quad , \boldsymbol{y}_i = t_i \sum_{j=1}^{k} t_j + m t_i \qquad \text{(22)}$$

$$\boldsymbol{\mu}_{\boldsymbol{\Delta}_i} = \boldsymbol{E}_i \, vec(\boldsymbol{\Delta}_i) \qquad , \quad \boldsymbol{E}_i = \begin{bmatrix} \sqrt{\alpha}\left(\boldsymbol{A_i^T} \otimes \boldsymbol{B_i}\right) \\ \sqrt{\alpha}\, MAT\left[\left\{\left(\boldsymbol{A_j^T} \otimes \boldsymbol{B_i}\right)\right\}_{j=1\neq i}^{k}\right] \\ \boldsymbol{0}_{mt_i \times mn} \end{bmatrix} \qquad \text{(23)}$$

If $\boldsymbol{Z} = \begin{bmatrix} \tau_i & \widehat{\boldsymbol{\mu}_i^H} \\ \widehat{\boldsymbol{\mu}}_i & \boldsymbol{I} \end{bmatrix}$, $\boldsymbol{W} = [\boldsymbol{0}_{mn\times 1} \quad \boldsymbol{E}_i^H]$, $\boldsymbol{X} = vec(\boldsymbol{\Delta}_i)$, $\boldsymbol{Y} = \begin{bmatrix} -1 & \boldsymbol{0}_{1\times y_i} \end{bmatrix}$,

then using Lemma 1, the

relaxed version of (21) would be:

$$\mathcal{M}_i = \begin{bmatrix} \tau_i - \lambda_i & \widehat{\boldsymbol{\mu}_i^H} & \mathbf{0}_{1\times mn} \\ \widehat{\boldsymbol{\mu}}_i & \boldsymbol{I}_{y_i} & -\delta_i \boldsymbol{E}_i \\ \mathbf{0}_{mn\times 1} & -\delta_i \boldsymbol{E}_i^H & \lambda_i \boldsymbol{I}_{mn} \end{bmatrix} \succcurlyeq 0\,,\ \lambda_i \geq 0\ \ \forall i\,. \tag{24}$$

So, (18) can be recasted as

$$\min_{\boldsymbol{A},\boldsymbol{B},\{\tau_i,\gamma_i,\lambda_i\}_{i=1}^k} \left\{ \sum_{i=1}^k \tau_i - \gamma_i \,\middle|\, \begin{matrix} TxP \leq P \\ \bar{h}_i(\boldsymbol{A}) \geq \gamma_i \ \ \forall i \\ \mathcal{M}_i \succcurlyeq 0, \lambda_i \geq 0\ \ \forall i \end{matrix} \right\} \tag{25}$$

The ACS method is employed to solve (25). If **B** is assumed to be known and constant, problem (26), and if **A** is assumed be known and constant, problem (27) can be derived

$$\min_{\boldsymbol{A},\{\tau_i,\gamma_i,\lambda_i\}_{i=1}^k} \left\{ \sum_{i=1}^k \tau_i - \gamma_i \,\middle|\, \begin{matrix} TxP \leq P \\ \bar{h}_i(\boldsymbol{A}) \geq \gamma_i \ \ \forall i \\ \mathcal{M}_i \succcurlyeq 0, \lambda_i \geq 0\ \ \forall i \end{matrix} \right\} \tag{26}$$

$$\min_{\boldsymbol{B},\{\tau_i,\lambda_i\}_{i=1}^k} \left\{ \sum_{i=1}^k \tau_i \,\middle|\, \mathcal{M}_i \succcurlyeq 0, \lambda_i \geq 0\ \ \forall i \right\}. \tag{27}$$

We use the PCCP algorithm to solve (26). To do so, first we should rewrite $\bar{h}_i(\boldsymbol{A})$ as a real-valued function:

$$\widetilde{\boldsymbol{A}}_{\boldsymbol{i}} = \Xi[\boldsymbol{A}_i] \in \mathbb{R}^{2n\times 2t_i}\,, \widetilde{\boldsymbol{H}}_i = \Xi\big[\ddot{\boldsymbol{H}}_{\boldsymbol{i}}\big] \in \mathbb{R}^{2n\times 2n} \tag{28}$$

Where

$$\Xi[\boldsymbol{X}] = [\Xi_1, \Xi_2]$$

in which

$$\Xi_1 = [\Re\{\boldsymbol{X}\}^T, \Im\{\boldsymbol{X}\}^T]^T\ \ , \Xi_2 = [-\Im\{\boldsymbol{X}\}^T, \Re\{\boldsymbol{X}\}^T\ ]^T.$$

As we know

$$tr(\boldsymbol{A}_i \boldsymbol{A}_i^H) = \frac{1}{2} tr(\widetilde{\boldsymbol{A}}_i \widetilde{\boldsymbol{A}}_i^T)$$

therefore using (28) and (19), the real-valued form of $\bar{h}_i(\boldsymbol{A})$ would be:

$$\tilde{h}_i(\widetilde{\boldsymbol{A}}) = \tfrac{1}{2}\eta(1-\alpha)\Big[\sum_{j=1}^k tr\big(\widetilde{\boldsymbol{A}}_j \widetilde{\boldsymbol{A}}_j^T \widetilde{\boldsymbol{H}}_i\big) - \sqrt{2}\boldsymbol{\epsilon}_{\boldsymbol{i}} \parallel \widetilde{\boldsymbol{A}}_j \widetilde{\boldsymbol{A}}_j^T \parallel_F + 2m\sigma_n^2\Big] \tag{29}$$

where $\widetilde{\boldsymbol{A}} = [\widetilde{\boldsymbol{A}}_1 \cdots \widetilde{\boldsymbol{A}}_k]$. So, using (29) the problem of (26) is recasted as:

$$\min_{\boldsymbol{A},\{\tau_i,\gamma_i,\lambda_i\}_{i=1}^k} \left\{ \sum_{i=1}^k \tau_i - \gamma_i \,\middle|\, \begin{matrix} TxP \leq P \\ \tilde{h}_i(\widetilde{\boldsymbol{A}}) \geq \gamma_i \ \ \forall i \\ \mathcal{M}_i \succcurlyeq 0, \lambda_i \geq 0\ \ \forall i \end{matrix} \right\}. \tag{30}$$

To solve (30), we resort to use Algorithm 1 and the main problem of (25) is solved using Algorithm 2.

Algorithm 1 PCCP based Algorithm to solve (30)

1: Set $L \leftarrow 0$, and generate $\widetilde{\boldsymbol{A}}^0 = [\widetilde{\boldsymbol{A}}_1^0, \cdots, \widetilde{\boldsymbol{A}}_k^0]$ randomly

2: Repeat

3: using the following equation, find the first-order estimate of $\tilde{h}_i(\widetilde{\boldsymbol{A}})$ at $\widetilde{\boldsymbol{A}}^L$

$$\hat{h}_i(\widetilde{\boldsymbol{A}}) = \tilde{h}_i(\widetilde{\boldsymbol{A}}^L) + \nabla\tilde{h}_i(\widetilde{\boldsymbol{A}}^L)^T vec(\widetilde{\boldsymbol{A}} - \widetilde{\boldsymbol{A}}^L) \quad (31)$$

4: Solve the following problem

$$\min_{\boldsymbol{A},\{\tau_i,\gamma_i,\lambda_i\}_{i=1}^k} \left\{ \sum_{i=1}^k \tau_i - \gamma_i \;\middle|\; \begin{array}{c} TxP \le P \\ \hat{h}_i(\widetilde{\boldsymbol{A}}) \ge \gamma_i \quad \forall i \\ \mathcal{M}_i \succcurlyeq 0, \lambda_i \ge 0 \;\; \forall i \end{array} \right\} \quad (32)$$

5: Convert $\mathbf{A}$ to $\widetilde{\mathbf{A}}$

6: $\widetilde{\mathbf{A}}^{\mathbf{L+1}} \leftarrow \widetilde{\mathbf{A}}$

7: $L \leftarrow L + 1$

8: Until some conditions are met, like $L = L_{max}$, i.e., maximum iterations number or a small increment is seen for the objective function, i.e.,

$$\left|\Gamma^L - \Gamma^{L+1}\right| \le \beta\,, \quad \text{where} \quad \Gamma = \sum_{i=1}^k \tau_i - \gamma_i$$

Proof of the convergence Algorithm 1 is in [7].

Algorithm 2 Algorithm to solve (25)

1: Generate **B** randomly and put it in (30)

2: $j \leftarrow 0$

3: Repeat

4: Solve (30) using Algorithm 1 to find **A**

5: Put **A** in (27)

6: Solve (27), find **B** and put it in (30)

7: $j \leftarrow j + 1$

8: While $\left|\left(\sum_{i=1}^k \tau_i^B - \gamma_i\right)^j - \left(\sum_{i=1}^k \tau_i^B - \gamma_i\right)^{j+1}\right| \le \beta$

where τ_i^B is the objective value of (27)

Simulation Results

We assess the performance of our system with the following scenario. We assume $\forall i,\ t_i = t$. To continue the simulation process, we choose the following value for the parameter: $\beta = 10^{-4}$. This value is chosen so that Algorithms 1 have enough time to converge with a proper performance [14]. We have done 200 rounds of Monte-Carlo simulations and all the given graphs have been averaged over these runs.

In this simulation we want to evalute the sum of harvested energy and the sum of MSE afromentioned system by changing data uncertainty , set these prameters $n = 3, m = 2, k = 3, t = 2, P = 1$, $\eta = 0.7$ and $\alpha = 0.3$. We would test the system performance by three kinds of data uncertainty, smsll, medium and large, i.e., $\delta = 0.05, 0.25, 0.5$. The state of $\delta = 0$ is perfect CSI.

In Figure.1, the sum MSE performance of the proposed algorithm is shown. It is clear that when the SNR is small, the sum MSE is mostly due to poor signal conditions and thus, the uncertainty regime is not that important. At high SNR regimes, the more uncertainty is in the CSI data, so decreasing the system performance since the uncertainty would lead to more conservativeness about the real state of the CSI.

In Figure. 2 the sum of energy harvested for the imperfect CSI cases, the perfect CSI case and the MM method of [5] is shown. It is clear with decreasing the uncertainty CSI data, the sum of harvested energy is increased. Interestingly, the proposed method would better performance from the MM method even experiencing small to medium uncertainty regimes, especially at higher SNR regimes. The perfect CSI case is about $40-60$ percent better than the case with $\delta = 0.5$ and about 5 percent better than the case with small uncertainty. At high SNR regimes, our algorithm is harvesting about 40 percent more energy than the MM method even with data uncertainty regime $\delta = 0.05$.

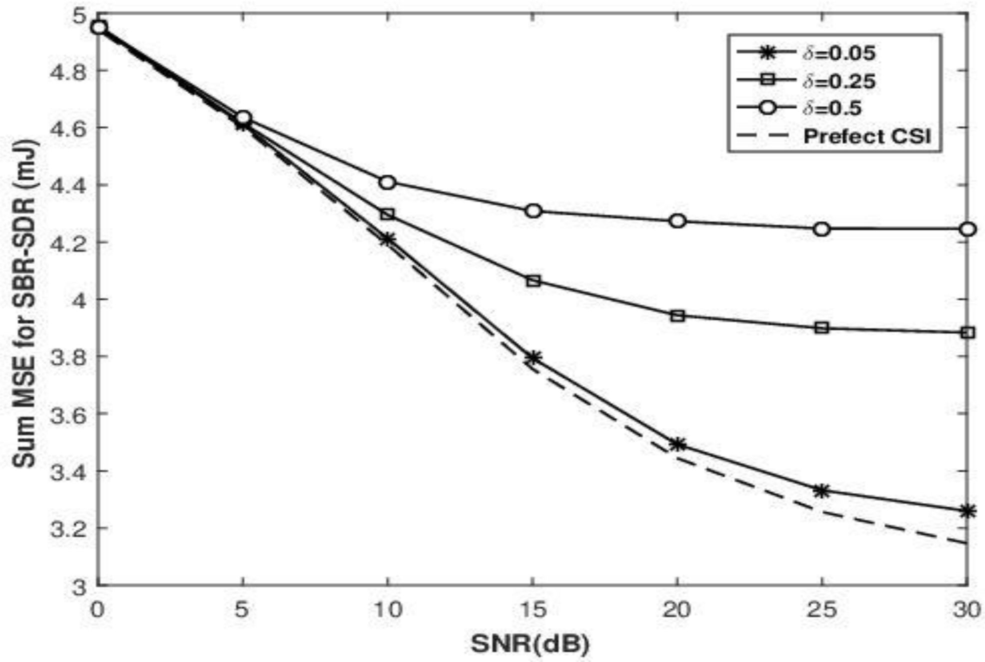

Figure 1: Change of Sum MSE by uncertainty size δ

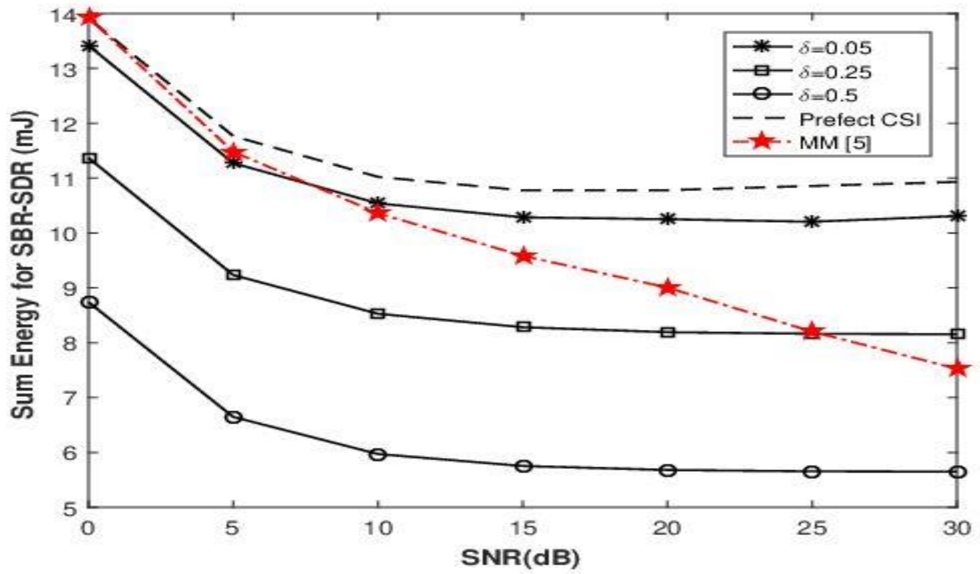

Figure 2 Sum energy harvested by changing uncertainty size δ

Conclusion

In this paper, a MU-MIMO SWIPT system in a BC configuration is examined . We do not restrict the number of users. Each receiver has power splitter and all of them are able to receive both power and information simultaneously. We jointly optimize the precoders and the decoders by maximizing the sum of harvested energy in EH modules and minimizing the sum MSE of symbol detection of all links concurrently with constraint on BS power under imperfect CSI. We can recast this problem into a DC problem, with function which is bi-convex. We employ SDR method to determine the worst case of the sum MSE and exploit SBR method to find the minimum the sum of harvested energy. We use a PCCP algorithm with a combination of an ACS algorithm to solve the problem. The simulation results confirm the efficiency of the proposed algorithm.

List of Symbols

The following notations and assumptions are used throughout the paper.

1. $(\cdot)^{\mathrm{T}}$, Transpose of a matrix or a vector
2. $(\cdot)^{\mathrm{H}}$, Hermitian of a matrix or a vector
3. $\|\cdot\|_{\mathrm{F}}^{2}$, Squared Frobenius norm of a matrix
4. $\mathrm{tr}(\cdot)$, Trace of a matrix
5. $\mathrm{vec}(\cdot)$, Vectorized matrix
6. $\mathbf{A} \otimes \mathbf{B}$, Kronecker product of matrices
7. $\mathrm{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, The distribution of a circularly symmetric complex Gaussian (CSCG) random vector with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$
8. $\mathbb{C}^{\mathrm{x} \times \mathrm{y}}$, The space of $\mathrm{x} \times \mathrm{y}$ complex-valued matrices
9. $\mathbb{R}^{\mathrm{x} \times \mathrm{y}}$, The space of $\mathrm{x} \times \mathrm{y}$ real-valued matrices
10. $\mathrm{MAT}[\{\{\mathbf{A}_{\mathrm{i}}\}_{\mathrm{i}=1}^{\mathrm{n}}\}]$, is used to show a tall matrix that is composed of the stacking of a series of indexed matrices, i.e., $[\mathbf{A}_1^{\mathrm{T}}, \cdots, \mathbf{A}_{\mathrm{n}}^{\mathrm{T}}]^{\mathrm{T}}$
11. $\mathbb{E}[.]$, Mathematical expectation of random variables
12. $\mathbf{I}_{\mathrm{k}}$, Identity matrix of size k
13. $\Re\{\cdot\}$ and $\Im\{\cdot\}$, are used, respectively to denote the real and imaginary parts of a complex number.
14. $\nabla \mathrm{h}$, Gradient of h

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